TENTAMEN NUCLEAR ENERGY

26-10-2015

14.00-17.00

A.Jacobshal 02

- This exam consists of 4 questions
- Value of the questions 1-4: 2, 2, 2.5, 2.5 points respectively
- The weight of each subquestion is the same
- Please use a separate sheet of paper for each question
- Please write your name and student number on each sheet of paper you hand in !!!
- Motivate your answers and give the full formulas on which your numerical answers are based

QUESTION 1.

- a) Calculate the energy (in eV) of thermal neutrons. How are they produced in a nuclear reactor?
- b) What is the energy (in eV) of fast neutrons? How are they produced in a nuclear reactor?
- c) What is the nuclear fuel in a (common type) fast nuclear breeder reactor, and why?
- d) What is the cooling medium for such a reactor, and why?
- e) What can you say about the moderator of such a reactor?
- f) The 6- or 4-factor formula for "k" has a simple form for a fast reactor. What is this formula for k_{∞} , and what is (approximately) the numerical value for its components

QUESTION 2.

Consider the nuclear fusion reaction $D + T \rightarrow {}^{4}He + n$

- a) Calculate the energy (in eV) that is released by the "mass defect" for this reaction.
- b) Calculate the energy (in J) that is released when 1 mole of deuterium fuses with 1 mole of tritium.

in case you did not find answer a), use 10^6

c) Calculate the amount of tritium used by a 5 GW (electric) fusion power plant for 3 years. Assume that the efficiency of the power plant is 1/3, and that the power plant is operational under the stated conditions for 90% of the time. in case you did not find answer b), use 10^{13}

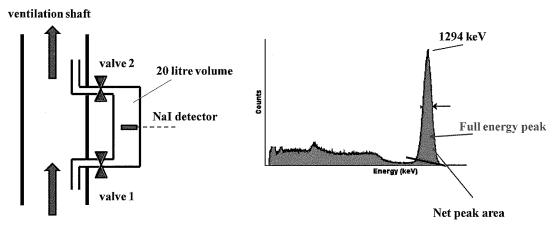
Data that can be used:

Boltzmann constant $k = 1.38 \times 10^{-23}$ J/K, electron charge $e = 1.6 \times 10^{-19}$ C Avogadro number $N_A = 6.023 \times 10^{23}$, speed of light $c = 3 \times 10^8$ m/s Atomic mass unit $1u = 1.66044 \times 10^{-27}$ kg Nuclear masses for n, 2 H, 3 H, 4 He: resp. 1.0086654, 2.0135536, 3.0155010, 4.0015062 u

QUESTION 3.

⁴¹Ar in the gaseous effluents from a nuclear reactor

Air contains a small amount of ⁴⁰Ar (stable). Due to neutron irradiation in a nuclear reactor part of this stable argon is converted to unstable ⁴¹Ar. The amount of ⁴¹Ar that is released to the environment through the ventilation shaft of a nuclear reactor is monitored with an NaI gamma spectrometer. The measurement set up consist of a fixed volume of 20 litres in which the NaI crystal of the spectrometer is positioned. Part of the exhaust air is bypassed via this volume (see figure below left).



The efficiency of the measurement set up is determined by first producing a ⁴¹Ar source by neutron irradiation of a small glass vial filled with ⁴⁰Ar. After the neutron irradiation this vial contains 400 MBq ⁴¹Ar.

- a) Calculate the dose a laboratory worker receives if he manipulates the ⁴¹Ar source directly after irradation for 30 seconds at 0.5 meter distance without using shielding. The glass vial may be approximated by a point source.
- b) Calculate the reduction of the dose found under a) if the worker uses a 4 cm thick lead shield between his body and the source. Build up may be neglected in this calculation.

An hour after the irradiation the vial is emptied in the 20 litres volume (valves 1 and 2 are closed) and a gamma spectrum (see figure above, right) is measured in 20 s. The number of counts in the net peak area of the full energy peak is $4 \cdot 10^7$.

- c) Calculate the efficiency (in cps per MBq m⁻³) of the measurement set up for detecting ⁴¹Ar.
- d) During normal reactor operation the valves 1 and 2 are open and a net count rate of 1 cps is measured. Calculate the ⁴¹Ar activity concentration (in Bq m⁻³) in the exhaust air. *In case you did not find the efficiency under d) use 292 cps per MBq m*⁻³.

- e) Assume that in the reactor building the air has a constant 41 Ar activity concentration of 30 kBq m⁻³. Calculate the neutron flux J (in m⁻²s⁻¹) in the reactor building if the atomic cross section for the production of 41 Ar by neutron capture on 40 Ar is 0.5 barn
- f) Suppose a worker at the reactor would work for 4 hours per day and for 200 days per year in air that contains this ⁴¹Ar concentration. What would be the effective dose per year for this worker due to ⁴¹Ar.

Data that can be used:

Γ: Specific gamma-ray dose constant (at 1 m) for 41 Ar: 1.877·10⁻⁴ mSv m² h⁻¹ MBq⁻¹. T_1 : Half life of 41 Ar: 109 min.

⁴¹Ar emits one gamma-ray per decay with an energy of 1294 keV.

 μ : Linear attenuation coefficient of lead for 1294 keV gamma radiation: 0.67 cm⁻¹. The effective dose rate due to working in an atmosphere contaminated with 1 Bq m⁻³ of ⁴¹Ar is: $5.3 \cdot 10^{-9}$ Sv per day.

1 barn: 10⁻²⁸ m².

⁴⁰Ar concentration in air: 1.8·10²² atoms per m³.

QUESTION 4.

In a thermal reactor the radioactive isotope ¹³⁵Xe is produced as the decay product of the fission product ¹³⁵I. It disappears by decay or by capture of a thermal neutron. The ¹³⁵I disappears only by decay into ¹³⁵Xe. The thermal neutron fission and absorption cross sections and the yield of ¹³⁵I in the thermal neutron induced fission of ²³⁵U are given below.

The thermal neutron flux in the reactor is $\Phi = 6 \times 10^{13}$ cm⁻² s⁻¹ at nominal power.

- a) Give the differential equations that describe the rate of change of the number of ¹³⁵Xe and ¹³⁵I atoms. Indicate which physical process is associated with the various terms in the equation.
- b) What the is the effective halflife $(T_{1/2})$ of ^{135}Xe when the reactor is in equilibrium. Explain your answer.
- c) How does the amount of 135 Xe in the reactor change after it has been switched of suddenly. What does this change imply for the k_{∞} of the reactor and for its behaviour if the power would be raised to the same level again after a relatively short period (not long relative to the halflifes of 135 Xe and 135 I). Sketch the time dependence of the amounts of 135 Xe and 135 I in the reactor in a graph. Motivate your answers and the graph.

Data that can be used: see other side

- halflife 135 Xe $T_{1/2}=9$ hours halflife 135 I $T_{1/2}=6.5$ hours 1 barn = 10^{-24} cm² yield of 135 I in thermal neutron induced fission of 235 U: b=0.0293 thermal neutron absorption cross section 135 Xe $\sigma_{a,135Xe}=2.66\times10^6$ barn thermal neutron absorption cross section 235 U $\sigma_{a,235U}=684$ barn thermal neutron fission cross section 235 U $\sigma_{f,235U}=585$ barn

1 a) E=kT=0.025 eV for T=(273+20) K
by produced after moderation
b) 1-2 MeV, products of Rician reaction
c) 238U (+239 Pu)
d) liquid metals, in particular Na
e) no moderation
f) k=nf &=1, p not relevant for fact reactors
considered lass safe than thermal reactors
(operational parameters, Na; reprocessing; weapons)
2 or) mass n = 1.0086654 y D 2.0135536 y
4 He= 4.0015062 3H 3.0155010
5,0101716 5,0290546
difference 0.018883 u x 931 = 17-58 Mer
23) () 31
b) $(6.023 \cdot 10^{23})(17.58; 10^{6}) = 1,06.10^{21} \text{ eV}$ $\times 1,6.10^{-19} = 1,697.10^{12}$
* 1,6. 10 19 = 1,697.10 1
e) P = 5 GW
t= 3 year = 3.365, 24.3600 = 9,46.10 s
6 = 6, 33
21 (5.19) (9.4(1.7)
Pel= E E /t = > E = (5.10) (9,46.10) 143,16]
0,33
n=nr, of mol T;
$n = \frac{\text{Emax}}{\text{Empl}} = \frac{143.10^{16}}{1697.10^{12}} = 84.3 \text{ 10}^{4} \text{ mol}$
Company of the state of the sta
M=n.3.4 gran = 84,3.3.10 = 2529.10 = 2529 kg.
90% gentional: 0.9 x2529 = 2276 kg/



Solution:

a)

As 30 seconds is very short compared to the half life of the source, decay during the manipulation can be neglected.

The (unshield dose rate is):

$$\dot{D} = \frac{\Gamma A}{r^2} = \frac{1.877 \cdot 10^{-4} \cdot 400}{0.5^2} = 0.3 \text{ mSv h}^{-1}$$

The exposure is $8.3 \cdot 10^{-3}$ h, thus total dose is $0.3 \cdot 8.3 \cdot 10^{-3} = 2.5 \,\mu\text{Sy}$.

b)

The dose is reduced by a factor,

$$e^{-\mu x} = e^{-0.67 \cdot 4} = 0.069$$

c)

Measurement is done 1 hour after the production of the source, thus activity of the source at the time of measurement is:

$$A(t) = A(t = 0)e^{-\frac{\ln 2 t}{\frac{T_1}{2}}} = A(t = 0)\left(\frac{1}{2}\right)^{\frac{t}{T_1}} = 400 \cdot \left(\frac{1}{2}\right)^{\frac{60}{109}} = 400 \cdot 0.68 = 273 \text{ MBq}$$

Measurement takes only 20 seconds, so decay during measurement can be neglected.

Concentration of ⁴¹Ar in the measurement volume is:

$$\frac{273 \text{ MBq}}{20 \cdot 10^{-3}} = 13.7 \text{ GBq m}^{-3}$$

And the efficiency that was asked is the count rate divided by the concentration:

$$\frac{4 \cdot 10^7}{20 \cdot 13.7 \cdot 10^3} = 146 \text{ cps per MBqm}^{-3}$$

d)

Concentration is: $\frac{1}{146} = 6.8 \cdot 10^{-3} \text{ MBq m}^{-3} = 6.8 \text{ kBq m}^{-3}$

e)

In this situation of constant concentration the production rate (per volume) of ⁴¹Ar is equal to the decay rate (per volume) of ⁴¹Ar.

The production rate is: $J\sigma C_{Ar40}$ The decay rate is: λC_{Ar41}

The activity concentration of ^{41}Ar is 30 kBq m⁻³ or $\frac{30\cdot10^3}{\lambda}=30\cdot10^3\frac{109\cdot60}{\ln2}=2.8\cdot10^8 \text{ atoms m}^{-3}$

Thus,

$$J = \frac{\lambda C_{Ar41}}{\sigma C_{Ar40}} = \frac{\ln 2}{109 \cdot 60} \cdot \frac{1}{0.5 \cdot 10^{-28}} \cdot \frac{2.8 \cdot 10^8}{1.8 \cdot 10^{22}} = 3.3 \cdot 10^{10} \text{ m}^{-2} \text{s}^{-1}$$

t) Effective dose is: $30 \cdot 10^3 \cdot 5.3 \cdot 10^{-9} \cdot (4/24) \cdot 200 = 5.3 \text{ mSv}.$

$$a) \quad \begin{split} \frac{dN_{\text{Xe}}}{dt} &= -\lambda_{\text{Xe}}N_{\text{Xe}} - \Phi_{\text{n}}\sigma_{\text{aXe}}N_{\text{Xe}} + \lambda_{\text{l}}N_{\text{l}} \\ \frac{dN_{\text{l}}}{dt} &= -\lambda_{\text{l}}N_{\text{l}} + \Phi_{\text{n}}\sigma_{\text{f235}}b_{\text{l}}N_{\text{235l}} \end{split}$$

First equation:

first term: decay of 135Xe into 135Cs;

second term: absorption of a neutron in 135Xe;

third term: decay of 135I into 135Xe.

Second equation

first term: decay of 1351 into 135Xe

second term: production of 135I by fission of 235U

- b) In equilibrium the amounts of 135I and 135Xe are independent of time. This is the case when the reactor has been operating at constant power (= constant fission rate) for a period much longer than the halflifes of 135I and 135Xe . The effective decay constant of 135Xe is $\lambda_{eff} = \lambda_{Xe} + \Phi_n \, \sigma_{aXe} = 2.1 \times 10^{-5} + 1.6 \times 10^{-4}$; the effective halflife is therefore 1.06 uur.
- c) The fraction ¹³⁵Xe that decays into ¹³⁵Cs is λ_{Xe} / λ_{eff} = 0.116
- d) Immediately after switching of the reactor the amount of 135Xe will increase because there is nor removal of 135Xe through neutron capture anymore, while its production through decay of 135I continues. If the reactor is switched on again after a relatively short periode the reactivity will be lower because the probability of neutron capture in 135Xe has become larger relative to that of capture in 235U. This decrease in reactivity has to be compensated by pulling out the control rods by a certain amound. The amount of 135Xe will subsequently gradually decrease again to the amount before switching off. The time constant for the increase of 135Xe after switching off is determined by the decay constants of 135I and 135Xe, the time constant for the decrease of 135Xe after switching on again is mainly determined by the neutron capture.

HERTENTAMEN NUCLEAR ENERGY

01-02-2016

18.30-21.30

room 54.12.0031

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- Value of the questions 1-4: 2, 2.5, 2, 2.5 points respectively
- The weight of each subquestion is the same
- Please use a **separate paper** for each question
- Please write your name and student number on each paper you hand in
- Motivate your answers and give the full formulas on which your numerical answers are based

QUESTION 1.

Give the approximate numbers (exact numbers not needed) for a) and b)

- a) How many nuclear power plants are operational worldwide?
- b) What is their contribution (in %) to the global electricity production?
- c) Can we do without nuclear energy in the "transition period" (the coming decades) according to the energy need scenarios predictions?
- d) What type of reactors is mostly used for electricity production?
- e) What is the nuclear fuel used in these reactors? What is the enrichment factor? Explain (briefly) why enrichment is needed.
- f) In what type of reactors can natural uranium be used as nuclear fuel? Explain (briefly) why.

QUESTION 2.

Consider a homogeneous reactor, the core is a vessel filled with heavy water with suspended particles consisting of a fuel mixture of $^{233}\text{UO}_2$ (1.5%) and $^{232}\text{ThO}_2$ (98.5%). Given for ^{233}U : neutron multiplicity $\nu=2.49$, fission cross section $\sigma_f=525$ b, absorption cross section $\sigma_a=586$ b. For ^{232}Th : absorption cross section $\sigma_a=7.56$ b.

- a) Give the definition of the reproduction factor η .
- b) Calculate this factor η using the numbers given above.
- c) Argue that, for this reactor, the fast fission factor $\varepsilon=1$.
- d) Given that the thermal utilization f = 1 and the resonance escape probability p = 0.89 for this reactor, calculate the multiplication factor k, assuming an infinite size.

If you did not find an answer for question b), use $\eta = 1.4$

e) The neutron transport equation is

$$\frac{\partial n}{\partial t} = (k_{\infty} - 1)\Sigma_a \Phi + D\nabla^2 \Phi$$

in which n (m⁻³) is the neutron concentration, Φ is the neutron flux (m⁻²s⁻¹), Σ_a (m⁻¹) is the macroscopic absorption cross section, and D (m) is the diffusion coefficient.

Show that this can be reworked into $\nabla^2 \Phi + B^2 \Phi = 0$ for stationary conditions. Give an expression for B.

f) Assuming the reactor has a spherical shape and introducing the so-called diffusion length L by $L^2 = \frac{D}{\Sigma_a}$, calculate the critical radius $R = \frac{\pi}{B}$ for this reactor given that L = 128 mm.

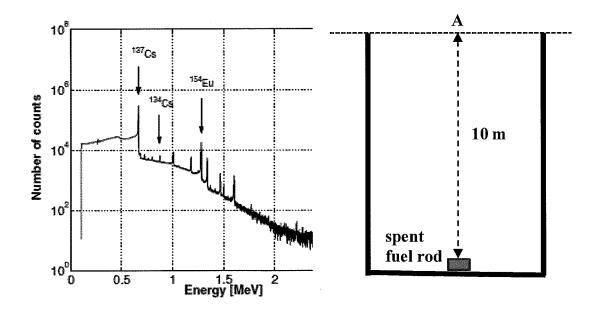
QUESTION 3.

In a homogeneous reactor the fuel is mixed with the coolant and moderator. This mixture can take different forms; it may be a solution but also a suspension of small fuel particles. The power is evacuated by passing the fuel- coolant mixture through a heat exchanger. In a heterogeneous reactor the fuel is separated from the coolant and moderator. Typically the fuel is located in a large number of fuel pins placed in the reactor core on a regular grid. The power is evacuated by passing the coolant through a heat exchanger.

- a) In which of these two types of reactors will the resonance escape probability p be the largest.
- b) How can the resonance escape probability be influenced in both types of reactors.
- c) In which of these two types of reactors will the prompt criticality margin be the largest.

QUESTION 4.

Spent fuel from nuclear reactors is highly radioactive. Water is good for both radiation shielding and cooling, so spent fuel is stored at the bottom of pools. A researcher uses a gamma spectrometer to measure a gamma spectrum (see figure below, left) of a very small sample (0.04 milligram) of the spent fuel taken from a 1000 kg spent fuel rod and concludes that ¹³⁷Cs is responsible for most of the activity.



To determine the activity of 137 Cs in the sample the researcher wants to use the 662 keV gamma-ray of 137 Cs. To find the efficiency for detection of this gamma-ray the researcher uses a calibration point source with a well-known 137 Cs activity of 3.01 kBq at 1-10-2000. The source is placed at 1-10-2015 on the spectrometer and after a measurement of 1 hour the net number of counts in the 662 keV full energy peak is $6 \cdot 10^5$. Thereafter the small fuel sample (that can be considered as a point source) is measured for 5 minutes and a net number of counts of $5 \cdot 10^6$ is collected in the 662 keV full energy peak.

- a) Calculate the efficiency (in cps per kBq) of the spectrometer for the 662 keV full energy peak.
- b) Calculate the ¹³⁷Cs activity in the spent fuel rod. *If you did not find the efficiency under a) use 164 cps per kBq.*

The spent fuel rod is lowered to the bottom of an empty spent fuel pool. The pool is 10 meters deep and so large that the spent fuel rod can be considered as a point source (see figure above, right).

c) Calculate the dose rate due to the ¹³⁷Cs activity in the spent fuel rod at the position A (see figure above, right) at the surface of the pool directly above the spent fuel rod. *If* you did not find the ¹³⁷Cs activity under b) use 10.2 TBq.

The spent fuel pool is now completely filled with water.

d) Calculate the dose rate in position A for the water filled pool. Take build up into account.

After having calculated the dose rate at the surface of the water-filled pool the researcher concludes that it is safe to swim at the surface of the pool with the spent fuel rod at the bottom.

e) Do *you* think this is safe, if you only consider the dose received due to gamma radiation of the spent fuel rod?

Suppose the spent fuel rod leaked some ¹³⁷Cs and during swimming the researcher accidently drinks 0.5 liter of the water having a ¹³⁷Cs concentration of 3 kBq per liter. The researcher also receives a dose to the skin of 5 mGy due to the beta radiation of ¹³⁷Cs.

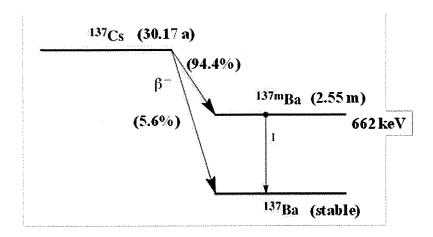
f) Calculate the effective dose that the researcher receives.

Please turn page for data that can be used!

Data that can be used:

Γ: Specific gamma-ray dose constant (at 1 m) for 137 Cs: $1.1 \cdot 10^{-4}$ mSv m² h⁻¹ MBq⁻¹. $T_{\frac{1}{2}}$: Half life of 137 Cs: 30.17 year. μ : Linear attenuation coefficient of water for 662 keV gamma radiation: 0.083 cm⁻¹. B: Build up factor for 662 keV gamma radiation and 10 m water: 1000. $e_{ing}(50)$: effective dose coefficient for the ingestion of 137 Cs: $1.3 \cdot 10^{-8}$ Sv Bq⁻¹.

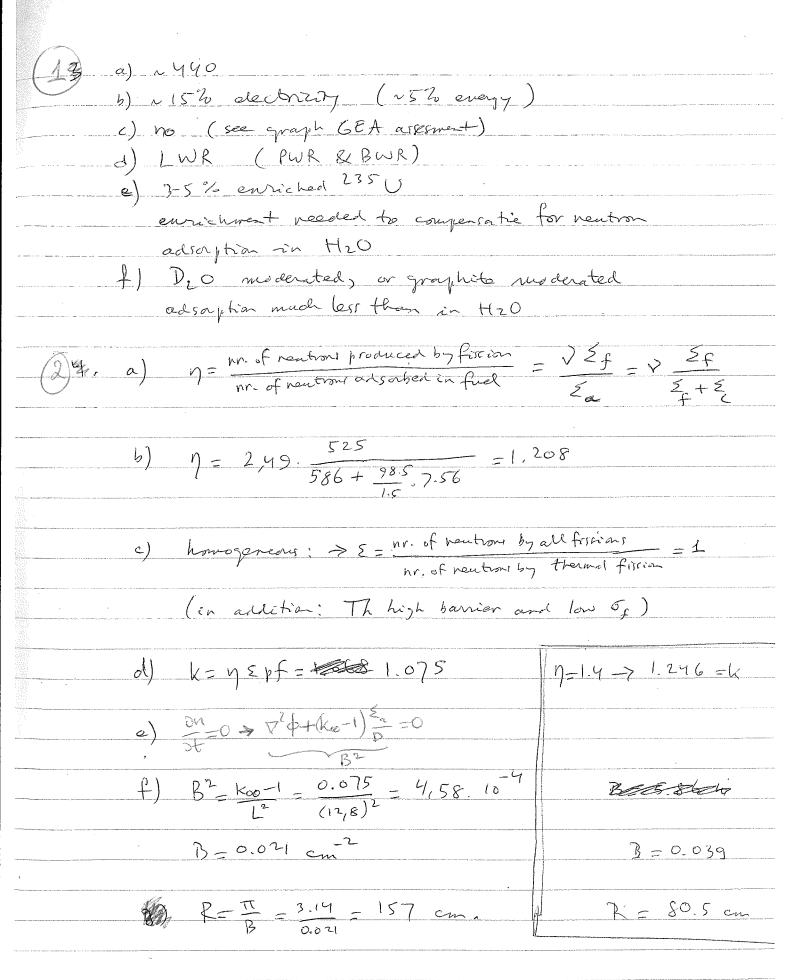
Simplified decay scheme of ¹³⁷Cs:



Tissue and radiation weighting factors:

	Weighting factors, w _T
Gonads	0.20
Red bone marrow	0.12
Colon	0.12
Lung	0.12
Stomach	0.12
Bladder	0.05
Breast	0.05
Liver	0.05
Oesophagus	0.05
Thyroid	0.05
Skin	0.01
Bone surface	0.01
Remainder	0.05

Type of radiation, R	Energy range	Quality or weighting factor, wR
Photons, electrons	All energies	l
Neutrons	<10 keV	5
	10-100 keV	10
4-15	100 keV-2 MeV	20
	2-20 MeV	10
	>20 MeV	
Protons	<20 MeV	5
Alpha particles, fission		
fragments, heavy nuclei		20





- a. The resonance escape probability p will be the largest in a heterogeneous reactor. The reason for this is that the moderation takes place in a medium that does not contain 238U. Therefore the neutron will less frequently encounter a 238U nucleus during the slowing down and therefore the probability that it will be absorbed in the energy region where the resonances are located is smaller.
- b. In a heterogeneous reactor the resonance escape probability can be influenced by the spacing between the fuel rods (the larger the distance between the fuel rods the larger the resonance escape probability) and by the enrichment (the larger the enrichment the larger the resonance escape probability; however, at the low enrichments used in power reactor this effect is minor)
 In a homogeneous reactor only the enrichment can be used to influence the resonance escape probability
- c. The prompt criticality margin is largest in a heterogeneous reactor because all the delayed neutrons are emitted in the core of the reactor, while in a homogeneous reactor a certain fraction of the delayed neutrons is emitted in the cooling circuit, where the neutron losses are for geometrical reasons larger than in the core of the reactor.



Solution:

a)

First we determine the activity of the point source at the time of the measurement which is 15 years after the date when the activity of the source is specified.

Using the half life from the decay scheme (30.17 year) we find for this activity A:

$$A = A_0 e^{-\lambda t} = A_0 e^{\frac{-\ln 2 t}{T_{\frac{1}{2}}}} = A_0 \left(\frac{1}{2}\right)^{\frac{t}{T_{\frac{1}{2}}}} = 3.01 \left(\frac{1}{2}\right)^{\frac{15}{30.17}} = 2.13 \text{ kBq}$$

The efficiency ε follows from (also account for the branching ratio):

$$\varepsilon = \frac{600000}{1 \cdot 3600 \cdot 0.944 \cdot 2.13 \cdot 10^3} = 0.082 \text{ cps per Bq} = 82 \text{ cps per kBq}$$

b) The 137 Cs activity A_{sample} in the sample (0.04 mg) is:

$$A_{sample} = \frac{\left(\frac{5000000}{300}\right)}{82} = 20.3 \text{ kBq}$$

The activity in the 1000 kg spent fuel rod is:

$$A_{rod} = \frac{20.3 \cdot 10000}{0.04 \cdot 10^{-3}} = 5.1 \cdot 10^9 \text{ kBq} = 5.1 \text{ TBq}$$

c)

Use,

$$\dot{D} = \frac{\Gamma A}{r^2} = \frac{1.1 \cdot 10^{-4} \cdot 5.1 \cdot 10^6}{10^2} = 5.6 \text{ mSv h}^{-1}$$

d)

Use,

$$\dot{D} = B \frac{\Gamma A}{r^2} e^{-\mu x} = 1000 \cdot 5.6 \cdot e^{-0.083 \cdot 1000} = 5600 \cdot 9 \cdot 10^{-37} = 5 \cdot 10^{-23} \text{ mSv h}^{-1}$$

eì

As the dose due to natural radiation is a few mSv per year, swimming seems to be save.

Amount of ¹³⁷Cs ingested is 1.5 kBq, this gives an effective dose of $1.5 \cdot 10^3 \cdot 1.3 \cdot 10^{-8} = 19.5 \,\mu\text{Sv}$.

The effective dose to the skin is: $0.01 \cdot 1 \cdot 5 \cdot 10^{-3} = 50 \,\mu\text{Sy}$.

Total effective dose: $19.5 + 50 = 69.5 \mu Sv$.